The Wess Zumino consistency condition: a paradigm in renormalized perturbation theory[1]

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1 Introduction

There is a jewel drowned within Julius' abundant scientific contribution:

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"Consequences of Anomalous Ward identities"

by

J. Wess (CERN); B. Zumino (CERN)

Phys. Lett. **B37** (1971) 95 [1]

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shortly followed by L.C. Biedenharn's[2] (University of Karlsruhe), talk at the Colloquium on group theoretical methods in Physics, Centre de Physique Théorique du CNRS, Marseille, 5-9 June 1972, whose author acknowledges discussions with Julius Wess and quotes Julius's lectures[3] on Anomalous Ward identities at the International Universität für Kern Physik, Schladming, 21 Feb.-4 March 1972.

This contains two parts:

a) the observation that current algebra anomalies, when occurring as right hand sides of Ward identities associated with a Lie algebra -in that case a gauge Lie algebra with structure group $(SU3)_L \times (SU3)_R$-, are constrained by "Wess Zumino consistency relations" which reflect the algebraic structure of the Lie algebra, in combination with the locality of (perturbative) field theory. This can be used to check the consistency of particular Feynman graph calculations.

L.C. Biedenharn's remark[2] was that these consistency relations were nothing else than Lie algebra cocycle conditions for the gauge Lie algebra under study.

b) the construction of a compensating action involving an element of the gauge group.

Part b) had a glamorous carreer thanks to E. Witten's[4] observation in 1984 of its ties with the topological properties of the gauge group, (besides its applications to physics already mentioned in the Letter). However interesting this is, we shall insist on the central role played by part a) within the known formulation of renormalized perturbation theory mathematically founded on causality-(locality)[5].

In section 2 we shall recall some of the main ingredients which go into the construction of renormalized perturbation expansions.

In section 3, we shall recall some of the standard or less standard examples which illustrate the importance of the WZCC (Wess Zumino Consistency Conditions).

Some further remarks are collected in the concluding section 4.

2 Renormalized perturbation theory: locality and power counting

Renormalized Perturbation Theory (R.P.T.) exists in essentially two distinct forms, both based on causality (locality) and power counting, the latter limiting the ambiguities which result from the singularity of local interactions.

In the first, more natural, set up [6][7], the building blocks are operators acting in some prescribed Fock space in correspondence with some quantized free fields. These operators are labelled by Wick polynomials of these free fields. The causal properties of the latter together with such general properties as translation covariance, positivity of the energy, relativistic covariance, result into a strict characterization of the ambiguities intrinsically attached to the definition of these operators, ("time ordered products" of Wick polynomials of the free fields) due to the distribution character which follows form the general principles[11].

In the second "functional set up[6][8], "time ordered products" are defined as functionals of some correspondingly labelled classical fields, and labelled by classical local polynomials of the latter. Necessary ambiguities are shown to be choosable within a restricted set, also characterized by locality and power counting although these limitations are not consequences of locality without some further assumptions[8].

The first, "on shell", (dispersive) point of view as well as the second, off shell, (Feynman) point of view are both treated in the classical article by H. Epstein, V. Glaser[6], a landmark in the long history of the role of causality in the construction of the renormalized perturbative series. Another long period[12] has finally given rise to a reexamination of these constructions, exemplified by two extensive studies, by M. Dütsch, F. Boas[7] and M. Dütsch, K. Fredenhagen[8], respectively, from which one can reconstruct the previous history.

This has a direct bearing on the definition of the physical content of these constructions[13] -notwithstanding all the limitations attached to the perturbative approach -since "physics" should be devoid of ambiguities.

Although less strictly founded, the off shell formalism[8] is in a much better shape that the more physical on shell formalism[7]. Thanks to the introduction of two further assumptions[8]

a) Time ordered products are multilinear functions of their arguments.

b) Time ordered products are restricted by the Action Ward identity (see section 3), one can organize the ambiguities within a renormalization group structure[14], which yields natural definitions of models and their physical contents.

The connection between the off shell and the on shell formalism is much less advanced.

To be brief, imposing the multilinearity of the operator time ordered products in their sets of Wick ordered local arguments, and the Hamiltonian Ward identity, the natural analog of the Action Ward identity (see section 3.2), is not proved at the moment compatible with Lorentz covariance, which is easy in the off shell formalism. Seen from an alternative point of view, a manifestly covariant set of on shell Ward identities has not yet been derived from the off shell formalism, which would ascertain the existence of an on shell renormalization group summarizing the known on shell ambiguities [14].

In spite of this incomplete knowledge, the role of Ward identities in the definition of the physical content of these constructions immediately comes to the forefront as their fulfillment results into a reduction of the renormalization group. In many interesting cases the possibility to fulfill a set of Ward identities proceeds directly through the construction of properly con-

strained sets of time ordered products via for instance the use of an intelligent regularization -renormalization procedure. Otherwise, the Wess Zumino consistency conditions (WZCC) are the only known general tool able to detect and or eliminate potential anomalies. This is tantamount to saying that the Wess Zumino consistency conditions are central in the definition of all models which are physically admissible in view of some anomaly cancellation conditions, namely, at the moment most of the physically interesting models[10].

Whereas the Wess Zumino consistency conditions (WZCC) appeared in the context of the description of the quotient of field space by the action of a Lie algebra -a gauge Lie algebra, in that case-, more general quotients have appeared since, -and will appear- giving rise to more general differential algebras which have been contemplated in the mathematical literature at great length since the fifties.

In section 3, we shall review a sample zoo of such structures mostly but not only originating from symmetries, which gives a modest measure of the impact of Phys. Lett. **B37** (1971) 95[1].

3 Examples

3.1 $SL2\mathbb{C}$ covariance (Group cohomology)

In conventional methods appealing to regularizations, it may happen that one needs to break Lorentz covariance (e.g. in the momentum space version of BPHZ[9]). Then, because $H^1(SL2\mathbb{C}, F) = 0$ for every finite dimensional representation space F of $SL2\mathbb{C}$, one can always recover covariance by the inclusion of suitable local counter-terms.

3.2 The Action Ward Identity[8]

In the off shell version where T products are labelled by n-uples of local classical polynomials $\mathcal{P}_i(\varphi)$ of fields φ and their derivatives, with values functionals of background field Φ, one wishes to impose

$$\partial_\mu^{(1)} \ T \ \mathcal{P}_1(\varphi)(x_1) \dots \mathcal{P}_n(\varphi)(x_n) = T \ \partial_\mu^{(1)} \ \mathcal{P}_1(\varphi)(x_1) \dots \mathcal{P}_n(\varphi)(x_n) \tag{1}$$

The proof that it is possible is given by M. Dütsch, K. Fredenhagen[8].

In the operator version where operator $\widehat{T}$ products are operators in Fock space labelled by local Wick polynomials of the free fields $\widehat{\varphi}$, one can similarly impose the Hamiltonian Ward identity

$$\overrightarrow{\partial}^{(1)} \ \widehat{T} : \mathcal{P}_1(\widehat{\varphi}) : (x_1) \dots : \mathcal{P}_n(\widehat{\varphi}) : (x_n) = \widehat{T} : \overrightarrow{\partial}^{(1)} \ \mathcal{P}_1(\widehat{\varphi}) : (x_1) \dots \mathcal{P}_n(\widehat{\varphi}) : (x_n) \tag{2}$$

where $\overrightarrow{\partial}$ denotes space derivatives. (For the on shell set up, see M. Dütsch, F.M. Boas[7]).

One could alternatively impose $SL2\mathbb{C}$ covariance (according to example 3.1), but one does not know at the moment whether this is compatible with the Hamiltonian Ward identity and allows for the absorption of the permissible ambiguities into an on shell renormalization group.

These identities have different equivalent meanings:

a) the scattering operator only depends on the action -resp. Hamiltonian-, not on the Lagrangian -resp. Hamiltonian-density.

b) (Energy) momentum conservation holds at each vertex of a renormalized Feynman graph.

This allows for the absorption of ambiguities into a renormalization group whose structure is based on the composition of formal power series (modulo the completion of our knowledge on the on shell formalism) :

3.3 Mass shell restriction

In the off shell formalism, one can restrict the ambiguities in such a way that T products with at least one argument in the ideal generated by the free fields equations of motion[7] belong to the corresponding ideal in the space of functionals. But this is in clash with the Action Ward identity.

It is a strongly supported but not yet proved conjecture that, in the on shell formalism, there is an $SL2\mathbb{C}$ covariant set of Hamiltonian Ward identities allowing for the absorption of ambiguities into an on shell perturbative renormalization group, of the following type: there is an $SL2\mathbb{C}$ invariant representative $\mathcal{W}$ of the Wick algebra as a subspace of $\mathcal{P}$, the algebra of local polynomials such that the Ward identity reads

$$\mathcal{D}^{x_1} \, T \, w_1(\varphi)(x_1)\ldots w_n(\varphi)(x_n) = T \, \mathcal{D}^{x_1} w_1(\varphi)(x_1)\ldots w_n(\varphi)(x_n) \tag{3}$$

for $w_i(\varphi) \in \mathcal{W}$, for all differential operators $\mathcal{D}^x$ such that $\mathcal{D} \, w_1(\varphi) \subset \mathcal{W}$ and such that T vanishes on shell whenever any of its arguments vanishes by virtue of the free field equations of motion, so that such T's define some $\widehat{T}$.

Remark: 3.2 and 3.3 are not related to symmetries.

3.4 Exact or softly broken global internal symmetries[15](abc)

The corresponding Ward identities can always be fulfilled for semisimple internal symmetry groups, again by $H^1(Lie \, G, F) = 0$ for semisimple $LieG$. If $U(1)$ factors occur, besides the Wess Zumino consistency conditions, renormalization group considerations are necessary.

3.5 Current algebra[15]

This is a sequel to 3.4.

The Wess Zumino consistency condition was solved for an arbitrary G, for renormalizable theories as early as 1975[15](e). That was a central part in the renormalization of quantized gauge theories. It took quite some time to release the power counting restrictions implied by renormalizability[16].

3.6 Gauge theories

This is a sequel to 3.5!

The Slavnov Taylor identity buries the gauge Lie algebra structure under an exotic layer of auxiliary fields whose role is to perform a sequence of quotients whereas locality is saved. There results a differential algebra whose relevant cohomology (H^0 and H^1) reduces to the corresponding part of the gauge Lie algebra cohomology.

The Faddeev Popov ghost field appears as the generator of the gauge Lie algebra cohomology.

The Faddeev Popov antighost together with a Nakanishi Lautrup multiplier field are associated with gauge fixing. Failing to include the latter results into a differential up to the antighost equation of motion, which is rather inconvenient[19].

Finally (this should actually come first!) the antifields are needed to express the fact that gauge invariance only holds up to the equations of motion.

The exotism of this collection of fields which allows to recover gauge invariance after its breaking through gauge fixing allows for the systematic use of the Wess Zumino consistency condition, pertaining to the Slavnov Taylor identity which is better (although not completely) understood now than when it first appeared.

3.7 "Field dependent" Lie algebras

All the preceding examples were associated with clearly defined algebraic structures encoded into a differential algebra. It is often not obvious how to reach such a clear situation. Here are a few more examples.

3.7.1 Gravity

The variables are taken as a vierbein e and a spin correction ω_{M_4} both locally defined one forms on space time M_4 and acted upon both by vector fields ξ on M_4 and $SL2\mathbb{C}$ gauge transformations Ω.

The structure equations for "parallel transport" read

$$\mathcal{L}(\xi, \Omega)e \ = \ i_\xi T + \Omega e + D_\omega(i_\xi e) \tag{4}$$

$$\mathcal{L}(\xi, \Omega)w \ = \ D_\omega \Omega + i_\xi R \tag{5}$$

where

$$T \ = \ de + \omega e, \quad R = d\omega + \frac{1}{2}[\omega, \omega] \tag{6}$$

$$D_\omega \ = \ d + t(\omega) \tag{7}$$

where t is the relevant representation of $\text{Lie}(SL2\mathbb{C})$ and

$$i_\xi = \text{contraction of form with vector field. } \xi \tag{8}$$

The commutation relations are

$$[\mathcal{L}(\Omega), \mathcal{L}(\Omega')] \ = \ \mathcal{L}([\Omega, \Omega']) \tag{9}$$

$$[\mathcal{L}(\Omega), \mathcal{L}(\xi)] \ = \ 0 \tag{10}$$

$$[\mathcal{L}(\xi), \mathcal{L}(\xi')] \ = \ \mathcal{L}([\xi, \xi']) + \mathcal{L}(i_\xi i_{\xi'} R) \tag{11}$$

where
$$[\xi, \xi'] = \text{commutator of vector fields } \xi, \xi'. \tag{12}$$

The second commutation relation (eq.10 might lead one to think that[18] at the infinitesimal level diffeomorphisms commute with $SL2\mathbb{C}$ gauge transformations, which is of course not right: the $\mathcal{L}$'s do not generate a Lie algebra because of the field dependent curvature term in the last commutator [eq.11].

What are then the Wess Zumino consistency conditions?

The answer is the following: if one lifts ξ along the horizontal planes of a background connection $\overset{0}{\omega}$ one produces a Lie algebra $\overset{0}{\mathcal{L}}$.

$$\left[\overset{0}{\mathcal{L}}(\Omega), \overset{0}{\mathcal{L}}(\Omega')\right] = \overset{0}{\mathcal{L}}([\Omega, \Omega']) \tag{13}$$

$$\left[\mathcal{L}^0(\xi), \mathcal{L}^0(\Omega)\right] = \mathcal{L}^0\left(\overset{0}{\mathcal{L}}_{(\xi)}\Omega\right) = \overset{0}{\mathcal{L}}\left(i_\xi D_{\overset{0}{\omega}}\Omega\right) \tag{14}$$

$$\left[\mathcal{L}^0(\xi), \mathcal{L}^0(\xi')\right] = \overset{0}{\mathcal{L}}([\xi, \xi']) - \dot{\mathcal{L}}\left(i_\xi i_\xi \overset{0}{R}\right) \tag{15}$$

where

$$D_{\overset{0}{\omega}} = d + t(\overset{0}{\omega}) \tag{16}$$

$$\overset{0}{R} = d\overset{0}{\omega} + \frac{1}{2}[\overset{0}{\omega}, \overset{0}{\omega}]. \tag{17}$$

$\overset{0}{\mathcal{L}}$ seems to depend on $\overset{0}{\omega}$. In fact, it does not: there is a Lie algebra $\mathcal{E}$, independent of $\overset{0}{\omega}$: one goes from $\overset{0}{\mathcal{L}}$ to $\overset{1}{\mathcal{L}}$ by the change of generators

$$\overset{0}{\mathcal{L}}(\xi) = \overset{1}{\mathcal{L}}(\xi) + \mathcal{L}(i_\xi(\overset{0}{\omega} - \overset{1}{\omega})) \tag{18}$$

$$\left(\overset{0}{\mathcal{L}}(\Omega) = \overset{1}{\mathcal{L}}(\Omega) = \mathcal{L}(\Omega)\right). \tag{19}$$

This, one knew beforehand: all objects sit on a bundle P. At the Lie algebra level, the automorphisms of the bundle are given by an extension of the diffeomorphisms of the base by gauge transformations

$$0 \to Lie\ \Omega \to Lie\ Aut\ P \overset{\overset{0}{\omega}}{\rightleftharpoons} Vect\ M \to 0 \tag{20}$$

A choice of $\overset{0}{\omega}$ defines (non canonically) a lift of Vect M into $\mathcal{E}$. This extension is neither central nor abelian.

Parallel transport is best defined within the cohomology algebra of $\mathcal{E}$ with values functionals of the vierbein and the spin connection by a change of generators

$$\Omega \to \Omega + i_\xi(\omega - \overset{0}{\omega}) \tag{21}$$

which is quite a licit operation whereas

$$\mathcal{L}^0(\xi) \to \overset{0}{\mathcal{L}}(\xi) + \mathcal{L}(i_\xi(\omega - \overset{0}{\omega})) \tag{22}$$

is quite strange since it mixes up parameters labelling the transformations with the field variables they act on[19].

3.7.2 Anomalies in the on shell operator formalism

Whereas the locality properties of the off shell formalism offer a good framework for the discussion of anomalies through the WZCC, it is much less so in the on shell formalism, from which there results algebraically hybrid situations which have been investigated in the case of free strings[20] (M. Kato, K. Ogawa, 1983, rigorized by I.B. Frenkel, H. Garland, G.J. Zuckerman, 1986)[20], where the result is exact, and by L.D. Faddeev[21] (1984) in the gauge case, at a semi-classical level.

In both cases, the anomaly manifests itself either at the gauge Lie algebra level as a field dependent cocycle which characterizes an extension of the gauge Lie algebra (in the string case, it is field dependent because the Faddeev Popov ghost is quantized), or, equivalently, as a lack of nilpotency of a BRST charge (both the definition of the charge and of its square require ad hoc renormalizations).

The situation is hybrid because the anomaly is a cocycle for the initial gauge Lie algebra.

As expected, the off shell version of this phenomenon is algebraically subject to the standard WZCC: it is simply the study of anomalies for the BRS current which can be shown to be an algebraic extension of the usual anomaly of the Slavnov identity alluded to in 3.6[22] (L. Baulieu, B. Grossmann, R. Stora, 1986).

Besides these algebraic formalities, it is worth mentioning that this class of examples can be described as an infinite dimensional version of 3.7.1: the base space is configuration space of classical fields acted upon by a gauge group, and above it a complex line bundle ("DET") (whose sections are functionals of the fields), with structure group $U(1)$ (phases). The anomaly is precisely of the form of the extra curvature term in Eq.11, whose locality in the field variables is related to general properties of index theorems[23].

3.8   A General Receptacle : BV

From the perturbative renormalization of gauge theories based on locality and power counting, we have learnt that besides gauge and matter fields we need:

- $\phi\pi$ ghosts, associated with infinitesimal gauge transformations,

- $\phi\pi$ antighost fields and Nakanishi Lautrup Lagrange multipliers, associated with gauge fixing

- "antifields", coupled to the non linear gauge transformations with parameter $\phi\pi$ ghost, and allowing their deformation -renormalization.

It was remarked by J. Zinn-Justin (1974) that writing the corresponding Ward identity (in this case called the Slavnov Taylor identity) for the vertex functional leads to its interpretation in terms of a graded symplectic -resp. contact- structure already met in mathematics in the context of Hochschild cohomology under the denomination "Gerstenhaber bracket", and further studied by I.V. Batalin, G.A. Vilkovsky (1981)[24].

The role of the antifields is to describe, with due respect of locality, symmetries which hold "modulo the equations of motion" ("on shell")[24][16]. This formalism has been subsequently applied to a variety of theories of the gauge type, including such exotic classes as supergravities, A. Zamolodchikov's W algebras, M. Konsevitch's quantization of Poisson structures, to quote only a few.

Needless to say, the WZCC have accordingly propagated into the study of the corresponding cohomologies.

4   Concluding Remarks

The innocent looking Wess Zumino consistency conditions have turned out to be an essential tool, at least -but not only- within the framework of renormalized perturbation theory, in the discussion of the physical content of field theory models founded on locality. There, restrictions of the ambiguities inherent to the singularity of local interactions experimentally proceed via the imposition of a variety of Ward identities and a discussion of their potential anomalies which, when present, precludes the use of any general intelligent regularization procedure. Various standard algebraic set ups have occured in field theory, beyond Lie algebras associated with symmetries, whose mathematics has been known for more than half a century, now, and together with them, a zoo of cohomologies for which the Wess Zumino consistency conditions are nothing else than cocycle conditions. Many more are expected to appear for the simple reason that in the sort of theoretical physics under practice, the enforcement of general principles -e.g. locality- delivers physical predictions as equivalence classes -resp. quotients- of objects belonging to a much larger class than that of the observables.

So, long life to WZCC!

References and Comments

[1] J. Wess, B. Zumino, Phys. Lett. **37B** (1971) 95.

[2] L.C. Biedenharn, in Colloquium on group theoretical methods in physics, Centre de Physique Théorique du CNRS, Marseille (F), 5-9 June 1972.

[3] J. Wess, proceedings International Universität für Kern Physik, Schladming (A), 21 Feb.-4 March 1972.

[4] E. Witten, Nucl. Phys. **B223** (1983) 422.

[5] This body of knowledge is globally referred to as the BPHZ framework.
Locality is most apparent in refs. [6], [7], [8] from which the past history can be recovered, but the combinatorics is most developed in Z (ref.[9]), summarized in [10].

[6] H. Epstein, V. Glaser, Ann. IHP **19** (1973) 211.

[7] M. Dütsch, F.M. Boas, Rev. Math. Phys. **14** (2002) 977.

[8] M. Dütsch, K. Fredenhagen, Rev. Math. Phys. **16** (2004) 1290,
M. Dütsch, K. Fredenhagen, in Symposium in honour of the 70th anniversary of Jacques Bros, CEA Saclay (2004).

[9] W. Zimmermann, Com. Math. Phys. **15** (1969) 208 ; Lectures on Elementary particles and Quantum Field Theory, Brandeis University, vol. I MIT Press, Cambridge Mass (1970).

[10] O. Piguet, S. Sorella, *Algebraic Renormalization*, Lecture notes in Physics **m28** Springer (1995).
Supersymmetric theories are treated in : O. Piguet, K. Sibold, *Renormalized Supersymmetry Birkhauser*, Boston 1986
E. Kraus, Ann. Phys. **262** (1998) 155
W. Holik, E. Kraus, M. Roth, C. Rupp, K. Sibold, D. Stockinger, Nucl. Phys. **B639** (2002) 3.

[11] R.F. Streater, A.S. Wightman, PCT, *Spin and Statistics and all that*, Benjamin, New York (1964)
R. Jost, *The General Theory of Quantized fields*, AMS, Providence, USA (1965).

[12] See for instance
G. Scharf, *Finite Quantum Electrodynamics*, Texts and Monographs in Physics, 2nd Edition Springer (1995)
G. Scharf, *Quantum Gauge Theories*, John Witey and Sons Inc. (2001) from which a good deal of that period can be reconstructed.
T. Hurth, K. Skenderis, Nucl. Phys. **B541** (PM) (1999) 566.

[13] The distinction between fields and observables is a leitmotiv reviewed in R. Haag, *Local quantum Physics*, Texts and monographs in Physics, 2nd Edition Springer (1996).
Its adaptation within renormalized perturbation theory is still under discussion. See, e.g. ref.[8].

[14] The ubiquity of our field theory knowledge which makes us oscillate between the operator formalism, where positivity properties are in principle more obvious, and the functional formalism in which Lorentz covariance is more easily seen to be compatible with locality, is part of theoretical life. Even in perturbation theory it is often not realized that renormalization is under control as demonstrated in ref.[6], also in the operator formalism.
The idea that ambiguities allowed by causality can be absorbed into a renormalization group goes back to

E.C.G. Stückelberg, A. Petermann, Helv. Phys. Act. **26** (1953) 499, in the off shell formalism.

The possibility to absorb the ambiguities into a renormalization group in the on shell formalism is still under investigation.

Its bearing on the definition of models and of the physical content of a model is foreseen in

W. Zimmermann, Com. Math. Phys. **97** (1985) 211

R. Oehme, W. Zimmermann, Com. Math. Phys. **97** (1985) 569.

For a recent review, geared towards applications see

M. Mondragon, G. Zoupanos, *Finite Unification at all-loops* mimeographed report (2000).

[15] a) B.W. Lee, Nucl. Phys. **B9** (1969) 649

b) J.L. Gervais, B.W. Lee, Nucl. Phys. **B12** (1969) 627

c) K. Symanzik, Com. Path. Phys. **16** (1970) 48 and in Cargèse lectures in Physics, Vol. **5** (1970), D. Bessis Ed., Gordon and Breach (1972)

d) C. Becchi, Comm. Math. Phys. **33** (1973) 97

e) C. Becchi, A. Rouet, R. Stora, CNRS Marseille preprint 1975, published in Field Theory, Quantization and Statistical Physics, E. Tirapegui Ed. D. Reidel Pub. Co. (1981).

[16] This history can be recovered from :

G. Barnich, F. Brandt, M. Henneaux, Phys. Rep. **338** (2000) 439 (hep-th-0002245)

G. Barnich, hep-th 001120 (2000)

G. Barnich, Local BRST cohomology in Yang Mills theory, mimeographed notes (2000).

[17] G.'t Hooft, Ed. *"50 years of Yang Mills"*, World Scientific, Singapore (2004).

[18] There is an amusing story going along with this, whose actors may remember.

[19] These are examples where difficulties to write down WZCC signal an improper -incomplete- algebraic setup. The following examples also bring some water to this mill. It is tempting to require as a principle that the Wess Zumino algebraic consistency be fulfilled in further constructions of that sort.

[20] M. Kato, K. Ogawa, Nucl. Phy. **212** (1983) 443

I.B. Frenkel, H. Garland, G.J. Zuckerman, Proc. Natl. Acad. Sci. USA **83** (1986) 8442.

[21] L.D. Faddeev, Phys. Lett. **B145** (1984) 81.

[22] C. Becchi, Nucl. Phys. **B304** (1988) 513

L. Baulieu, C. Becchi, R. Stora, Phys. Lett. **B180** (1986) 55

L. Baulieu, B. Grossmann, R. Stora, Phys. Lett. **B180** (1986) 95

M. Abud, J.P. Ader, J.C. Wallet, Ann. Phys. **203** (1990) 339.

[23] J. Mickelsson, *Current Algebra and Groups*, Plenum 1989.

[24] J. Zinn-Justin in *"Trends in Elementary Particle theory*, Bonn (1974), H. Rollnick, K. Dietz Eds., Lecture notes in Physics Vol. 37, Springer Verlag 1975

I.A. Batalin, G.A. Vilkovisy, Phys. Lett. **B102** (1981) 27

M. Gerstenhaber, Ann. Math. **78** (1963) 267.